Newtonian Mechanics

Kinematics

Imran Mirza (PGCC, University of StrathClyde, UK)
WhatsApp No. 00923007991600

Mechanics

The study of *Physics* begins with mechanics.

Mechanics is the branch of physics that focuses on the motion of objects and the forces that cause the motion to change.

There are two parts to mechanics: *Kinematics* and *Dynamics*.

Kinematics deals with the concepts that are needed to describe motion, without any reference to forces.

Dynamics deals with the effect that forces have on motion.

Introduction

Kinematics is the **science of describing the motion of objects using words, diagrams, graphs, and equations**.

The goal of kinematics is to develop mental models to describe the motion of real-world objects.

We will learn to describe motion using:
1. Words
2. Diagrams
3. Graphs
4. Equations

Describing Motion with words

The motion of objects can be **described by words**.

Even a person without a background in physics has a collection of words, which can be used to describe moving objects. For example, going faster, stopped, slowing down, speeding up, and turning provide a sufficient vocabulary for describing the motion of objects.

In physics, we use these words as the language of kinematics.
1. **Distance and Displacement**
2. **Speed and Velocity**
3. **Acceleration**

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These words which are used to describe the motion of objects can be divided into two categories.

The quantity is either a **vector** or **scalar**.

1. Scalars are quantities which are described by a magnitude only.

2. Vectors are quantities which are described by both a magnitude and a direction.

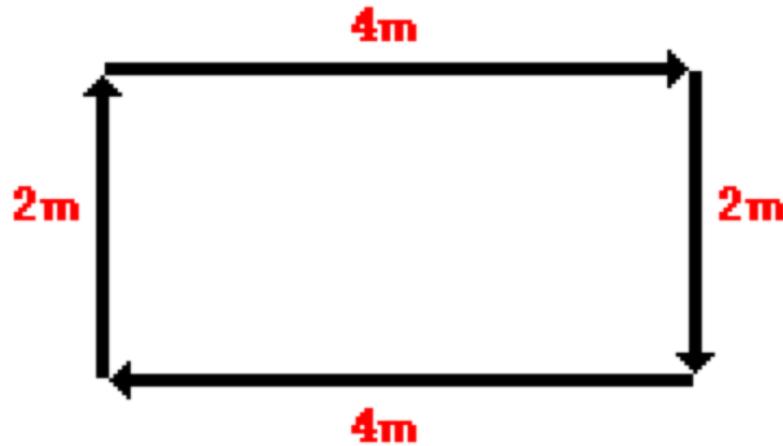| Distance	Displacement
Distance refers to **the total length of travel irrespective of the direction of the motion.**	Displacement refers to **the distance moved in a particular direction.** It is the object's overall change in position.
It is a **scalar quantity.** SI unit: metre (m) Other common units: kilometre (km), centimetre (cm)	It is a **vector quantity.** SI unit: metre (m) Other common units: kilometre (km), centimetre (cm)

Distance vs. Displacement

- You drive the path, and your odometer goes up (your distance).

- Your displacement is the shorter <u>directed</u> distance from start to stop (green arrow).
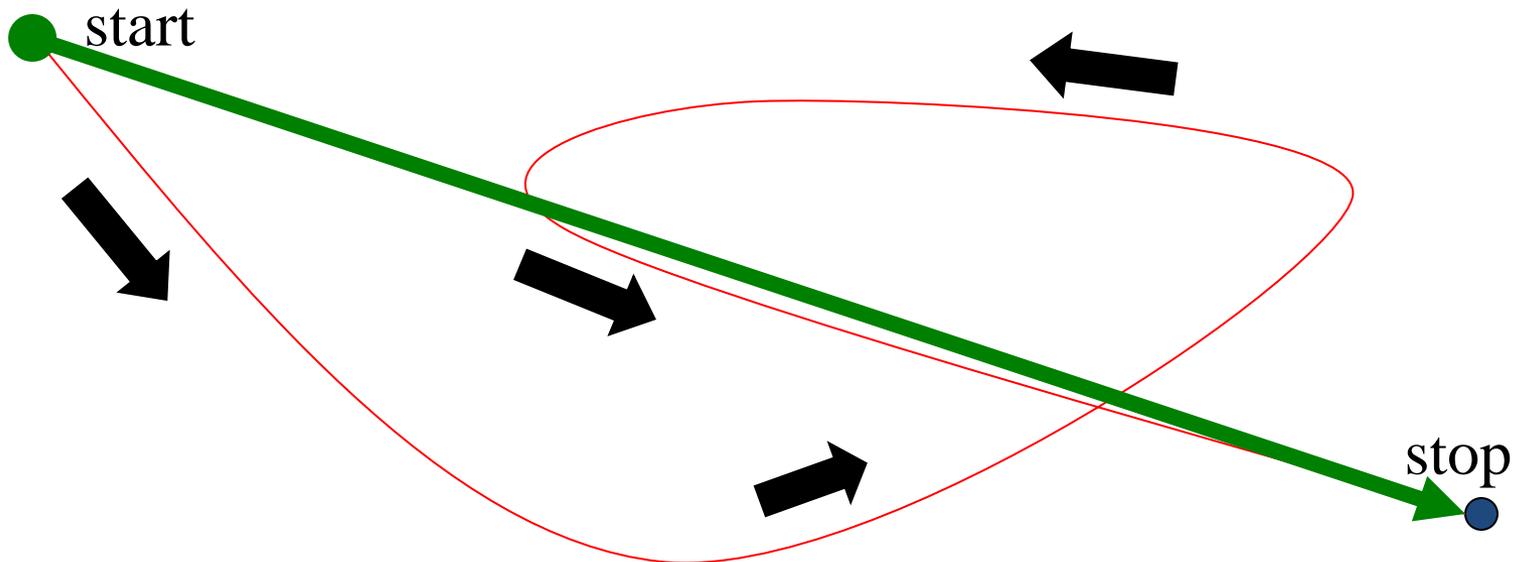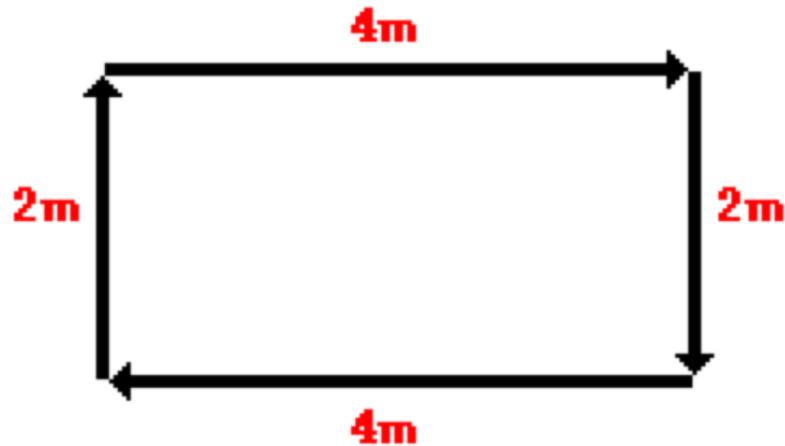
Example 1

A student walks 4 m East, 2 m South, 4 m West, and finally 2 m North.

Total distance = 12 m

During the course of his motion, the total length of travel is 12 m.

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Total displacement = 0 m

When he is finished walking, **there is no change in his position.**
The 4 m east is "canceled by" the 4 m west; and the 2 m south is
"canceled by" the 2 m north.

Speed	Velocity
Speed is **the rate of change of distance.** It is a **scalar quantity.** $$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$	Velocity is the **distance travelled in a specific direction.** It is also defined as **the rate of change of displacement.** It is a **vector quantity.** $$\text{Velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

When evaluating the velocity of an object, one must keep track of direction.

The direction of the velocity vector is the same as the direction which an object is moving. (It would not matter whether the object is speeding up or slowing down.)

For example:

If an object is moving rightwards, then its velocity is described as being rightwards.

Boeing 747 moving towards the west with a speed of 260m/s has a velocity of 260m/s, west.
Note that speed has no direction (it is a scalar) and velocity at any instant is simply the speed with a direction.

Instantaneous Speed and Average Speed

As an object moves, it often undergoes changes in speed.

The speed at any instant is known as the **instantaneous speed**. (From the value of the speedometer)
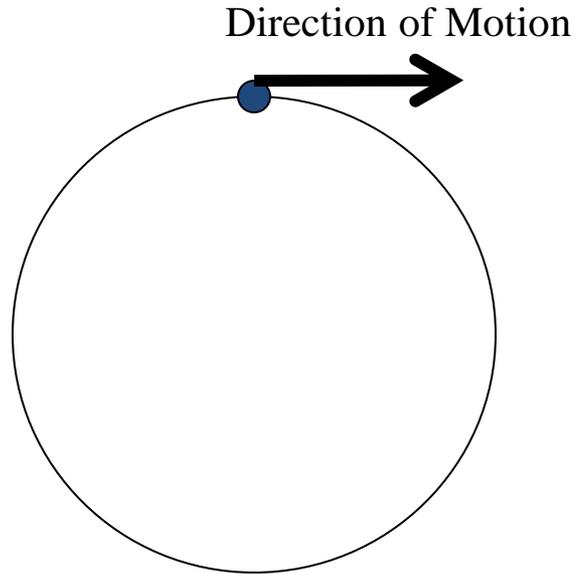

The average speed of the entire journey can be calculated:

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Speed Vs Velocity

An object is moving in a circle at a **constant speed** of 10 m s^{-1}.
We say that it has a constant speed but **its velocity is not constant.**
Why?

Direction of Motion

The direction of the object keeps changing.

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Acceleration

- An object whose velocity is changing is said to accelerate.
- If the direction and / or speed of a moving object changes, the object is accelerating
- Acceleration is the rate of change of velocity

Time (s)	Velocity (m/s)
0	0
1	10
2	20
3	30
4	40
5	50

Acceleration

Acceleration is a vector quantity

SI unit: ms^{-2}

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

where a = acceleration, v = final velocity, u = initial velocity and t = time.

$$a = \frac{v - u}{t}$$

Describing Motion with Graphs

1. Plot and interpret a distance-time graph and a speed-time graph.

2. Deduce from the shape of a distance-time graph when a body is:
    (a) at rest
    (b) moving with uniform speed
    (c) moving with non-uniform speed

3. Deduce from the shape of a Velocity-time graph when a body is:
    (a) at rest
    (b) moving with uniform speed
    (c) moving with uniform acceleration
    (d) moving with non-uniform acceleration

4. Calculate the area under a speed-time graph to determine the distance travelled for motion with uniform speed or uniform acceleration.

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Key Concepts

Distance-time Graph

Gradient of the Distance-time Graph is the speed of the moving object

Speed-time Graph

Gradient of the Speed-time Graph is the acceleration of the moving object.

Area under the Speed-time Graph is the distance travelled.

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Distance-time Graph

A car has travelled past a lamp post on the road and the distance of the car from the lamp post is measured every second. The distance and the time readings are recorded and a graph is plotted using the data. The following pages are the results for four possible journeys. The steeper the line, the greater the speed.
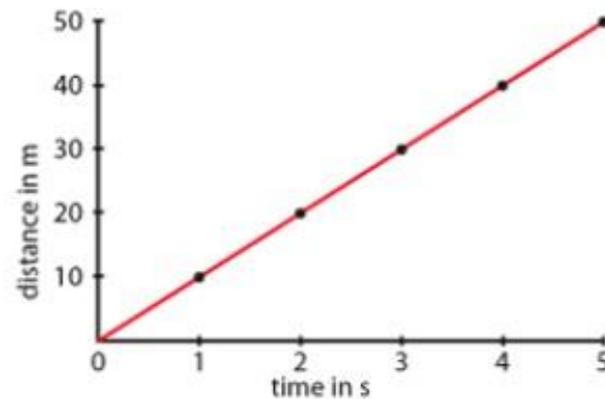
(a) Car at rest

Time in s	0	1	2	3	4	5
Distance in m	25	25	25	25	25	25
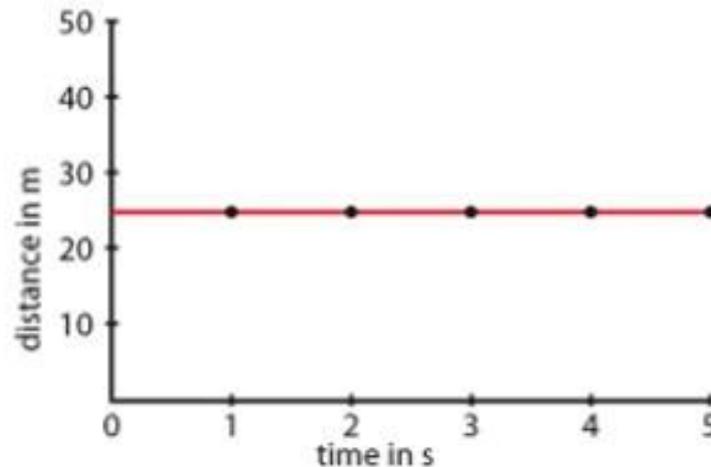
The car is parked 25 m from the post, so the distance remains the same.

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(b) Car moving with a uniform speed of 10 m s^{-1}

Time in s	0	1	2	3	4	5
Distance in m	0	10	20	30	40	50

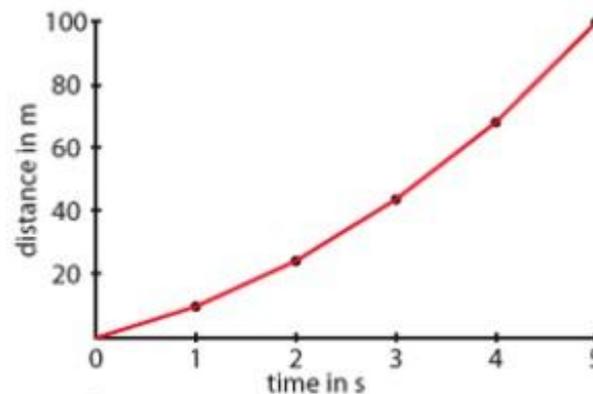

Distance increases 10 m for every 1 s.

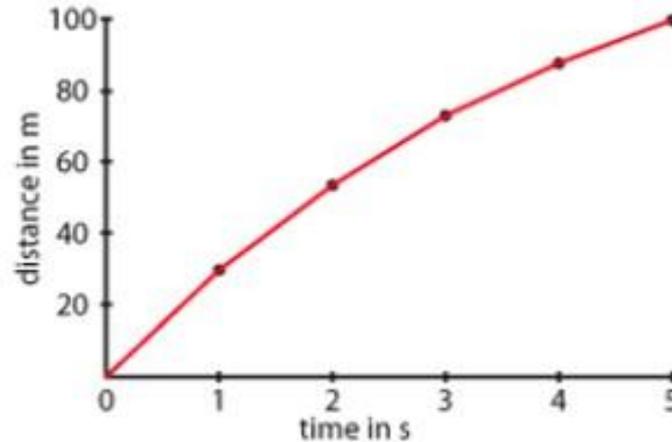(c) Car moving with non-uniform speed
   (i)   Car accelerating

Time in s	0	1	2	3	4	5
Distance in m	0	10	25	45	70	100

Speed increases, so the car travels a longer distance as time increases.

(ii) Car decelerating

Time in s	0	1	2	3	4	5
Distance in m	0	30	55	75	90	100

Speed decreases, so the car travels a shorter distance
as time increases.

The gradient of the distance-time graph gives the speed

of the moving object.

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Velocity - Time Graph

- The **gradient** of the velocity-time gradient gives a value of the changing rate in velocity, which is the **acceleration** of the object.

- The **area** below the velocity-time graph gives a value of the object's **displacement**.
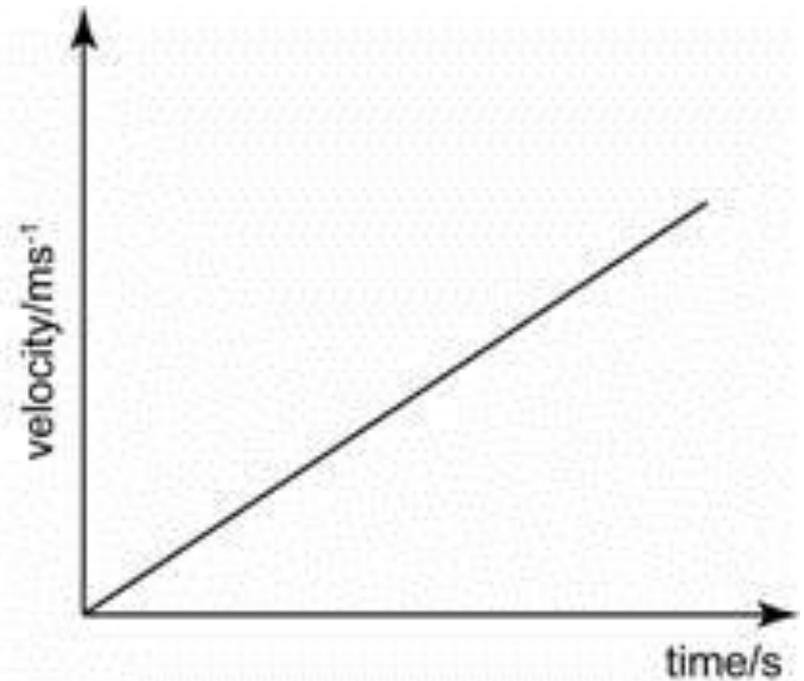
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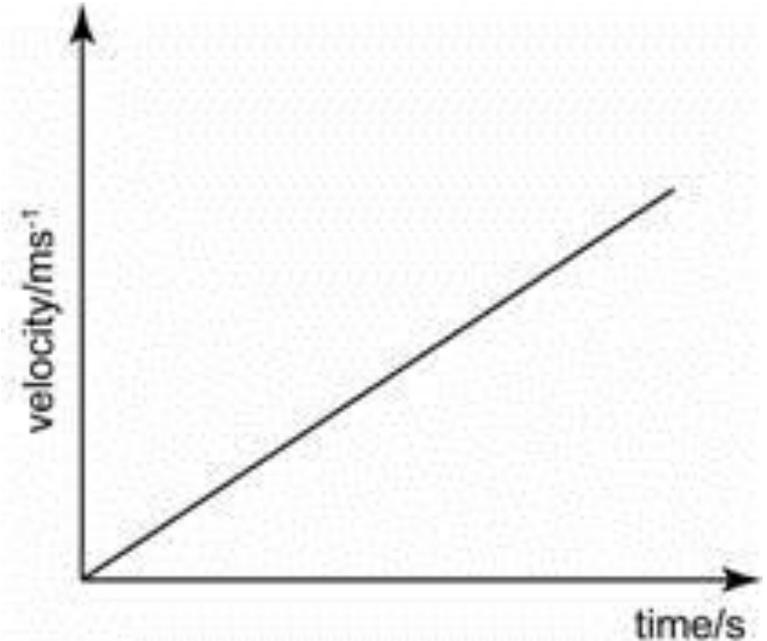
Analysing Velocity - Time Graph
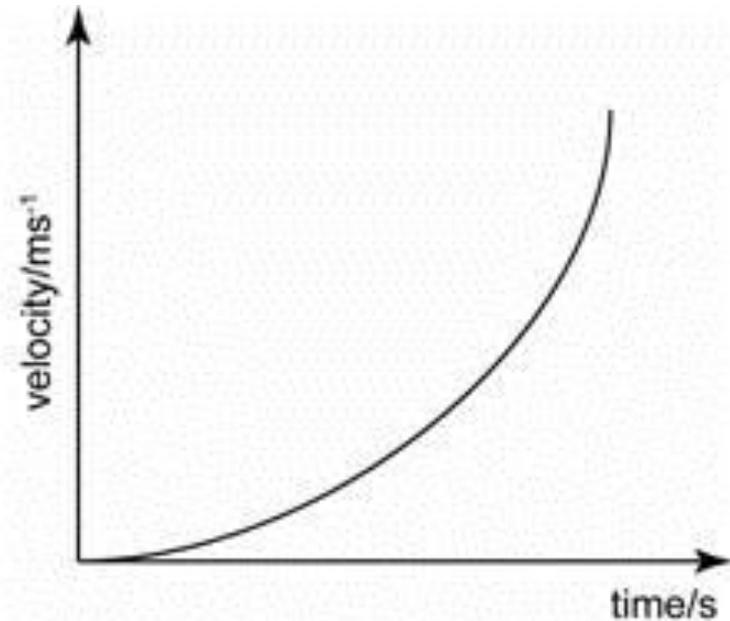
- ## Uniform Velocity
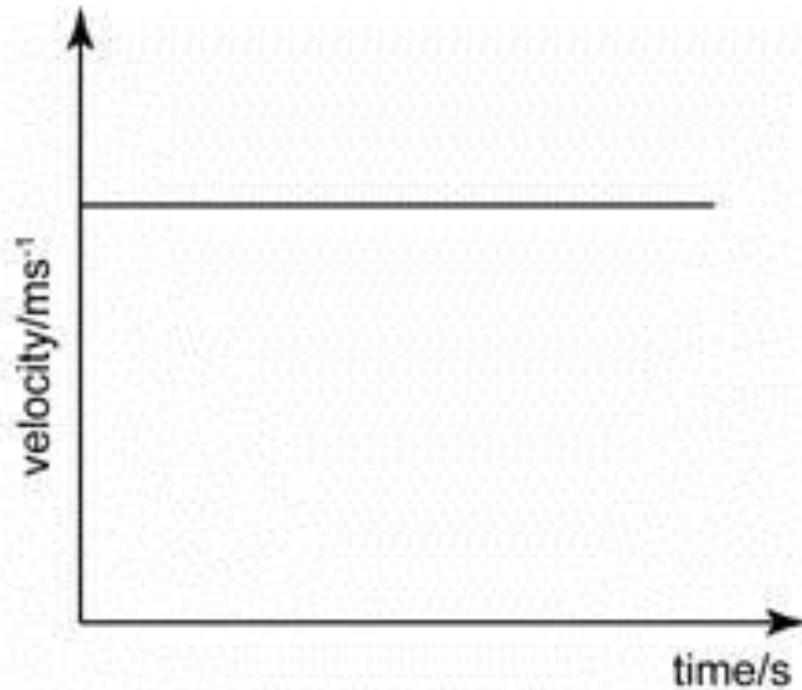
- ## Uniform Acceleration

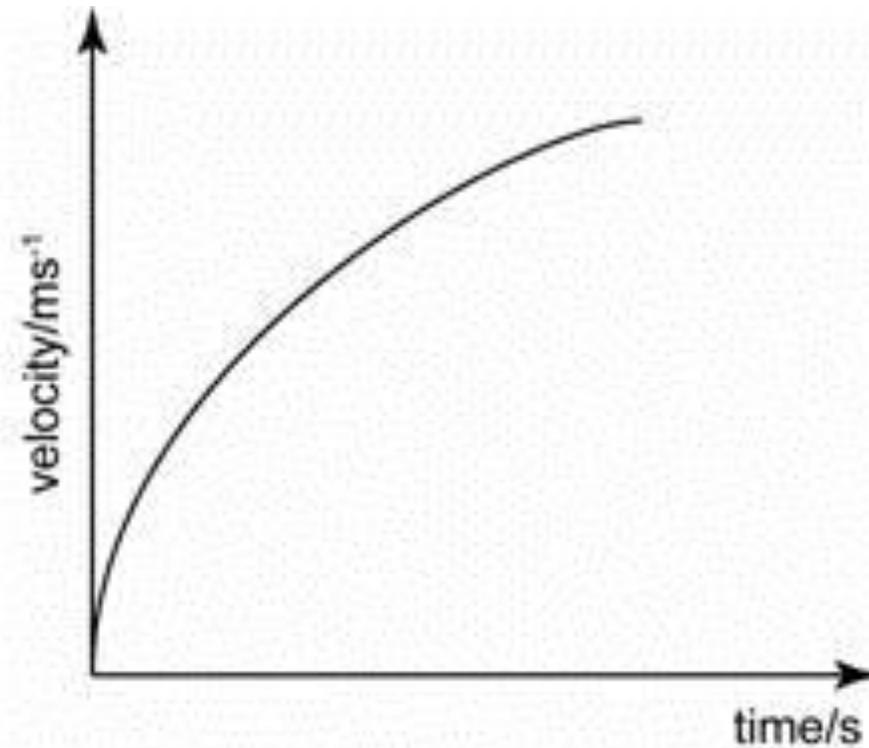# Analysing Velocity - Time Graph

- Uniform deceleration
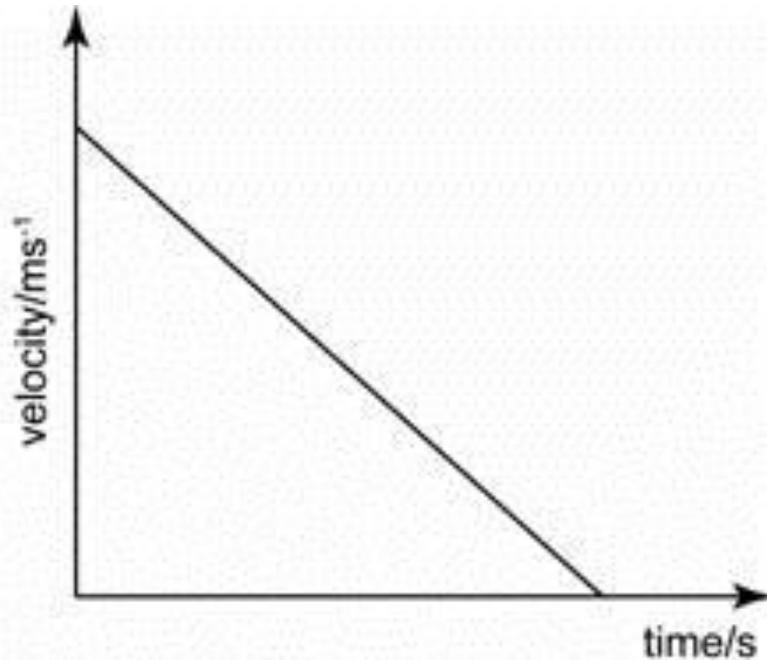
- Increasing acceleration

Analysing Velocity - Time Graph

Decreasing acceleration

How do you find the gradient of velocity-time graph?

➢ You need to select two points on the graph, for example (x_1, y_1) and (x_2, y_2).

➢ Once you have selected the points you put then into the equation $m = (y_2 - y_1)/(x_2 - x_1)$

➢ m = the gradient

• The gradient represents the acceleration.

• In other words, We take the vertical reading from the graph where the acceleration finishes and divide it by the horizontal reading where the acceleration finishes.

Example 1

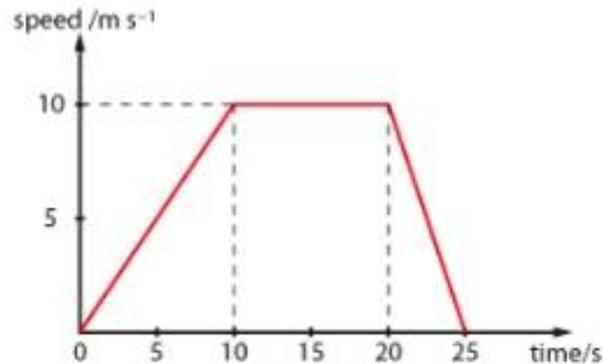Figure 2.14 shows the speed-time graph of a moving lift.

Figure 2.14

(a) What is the maximum speed of the lift?

(b) For how many seconds does the lift move?

(c) How much speed does the lift gain in the first 10 seconds? What is its acceleration?

(d) What is the deceleration of the lift in the last 5 seconds?

Solution

(a) The maximum speed of the lift is 10 m s^{-1}.

(b) The lift moves for 25 s.

(c) The lift gains 10 m s^{-1} in the first 10 s.

$$\text{Acceleration} = \frac{10 - 0}{10}$$
$$= 1 \text{ m s}^{-2}$$

(d) $\text{Acceleration} = \dfrac{0 - 10}{5}$
$$= -2 \text{ m s}^{-2}$$

Therefore the deceleration is 2 m s^{-2}.

Example 2

Figure 2.15 shows the speed-time graph for a journey of a boy from his house to school. Look at the shape of the graph and describe the type of motion in each stage.
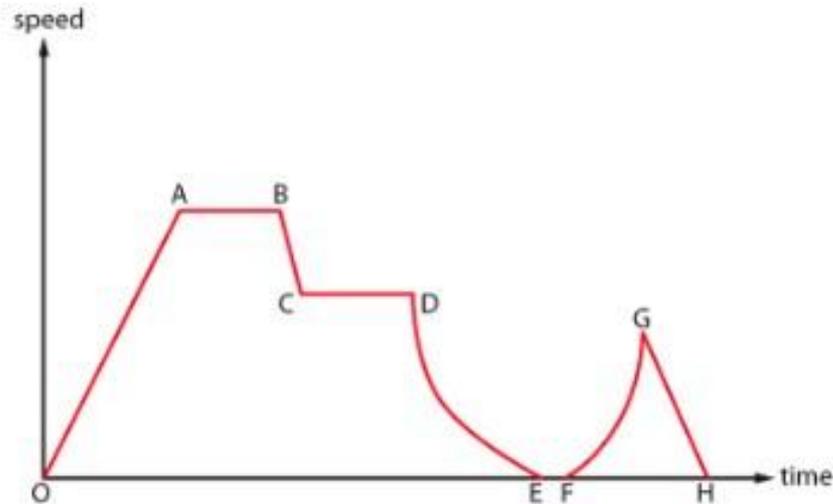

Figure 2.15

Solution

O	left home
O-A	moving with uniform acceleration
A-B	moving with uniform speed
B-C	moving with uniform deceleration
C-D	moving with uniform speed (speed lower than A-B)
D-E	moving with non-uniform deceleration (decreasing deceleration)
E-F	not moving
F-G	moving with non-uniform acceleration (increasing acceleration)
G-H	moving with uniform deceleration
H	reached school

Area under a speed-time graph

The figure below shows the speed-time graph of a car travelling with a uniform speed of 20 ms^{-1}. The distance travelled by the car is given by:

Distance = speed x time = 20 x 5

$$= 100 \text{ m}$$

The same information of distance travelled can also be obtained by calculating the area under the speed-time graph.

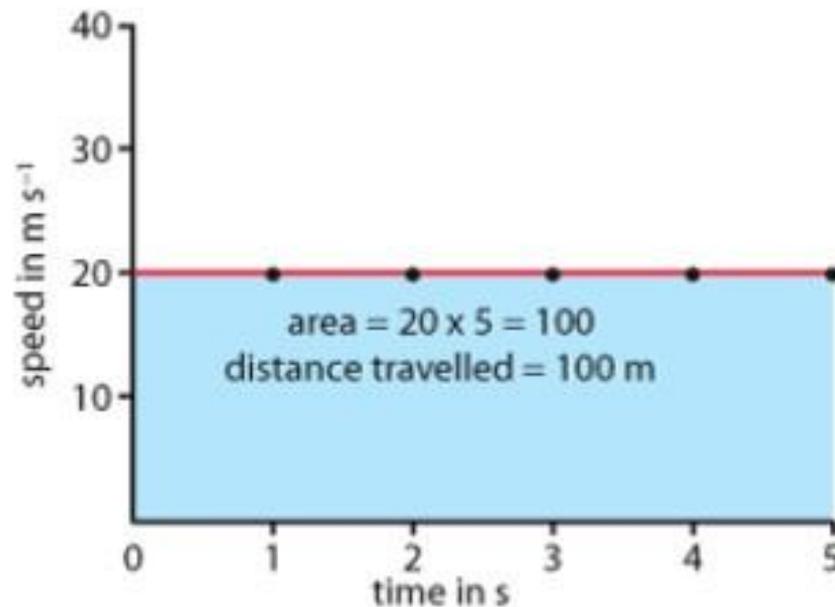The area under a speed-time graph gives the distance travelled.

Figure 2.16

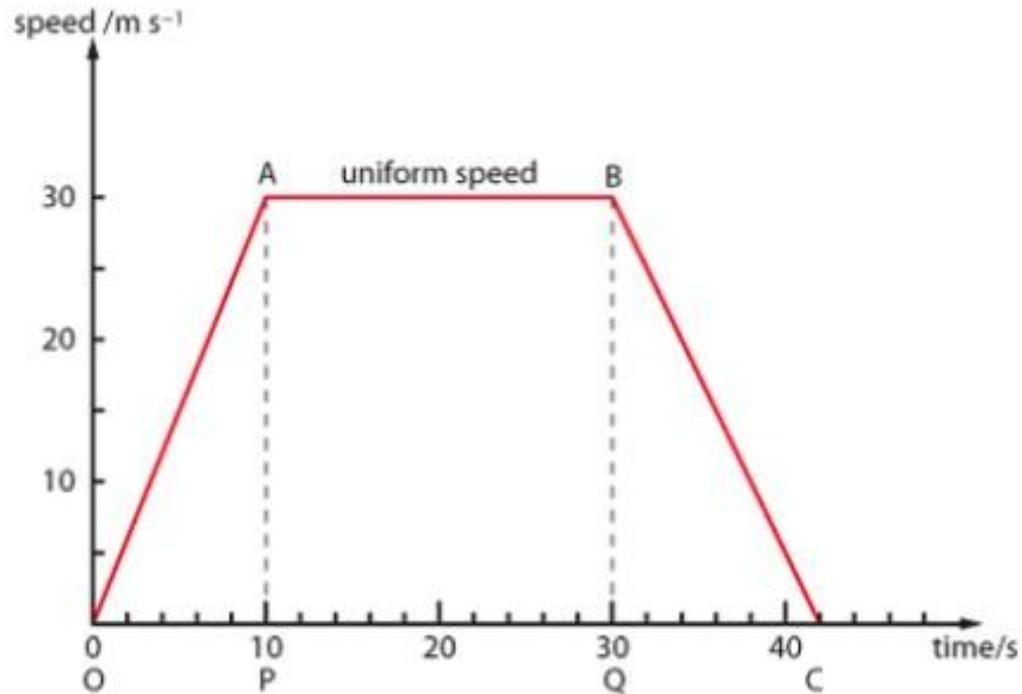# Example 3 - Question

speed /m s⁻¹

Figure 2.17 Speed-time graph of a car accelerating, moving with uniform speed and then decelerating

Figure 2.17 shows the speed-time graph of a car travelling along a straight road.

(a) What is the distance travelled during the first 10 s?
(b) What is the total distance travelled?
(c) What is the time taken for the whole journey?
(d) What is the average speed for the whole journey?

Example 3 - Solution

Solution

(a) During the first 10 s, distance travelled = area of triangle OAP

$$= \frac{1}{2} \times 10 \times 30$$

$$= 150 \, m$$

(b) Total distance travelled = area of trapezium OABC

$$= \frac{1}{2} \times (20 + 42) \times 30$$

$$= 930 \, m$$

(c) Time taken for the whole journey = 42 s

(d) Average speed for the whole journey $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{930}{42}$$

$$= 22.1 \, m \, s^{-1}$$

Fall freely…..

- http://www.youtube.com/watch?v=go9uekKOcKM
- http://www.youtube.com/watch?v=FHtvDA0W34I

Uniformly accelerated motion

- **Free fall** is motion with no acceleration other than that provided by gravity.

In other words........

- **A free-falling object is an object which is falling under the sole influence of gravity.**

- **Any object which is being acted upon only be the force of gravity is said to be in a state of free fall.**

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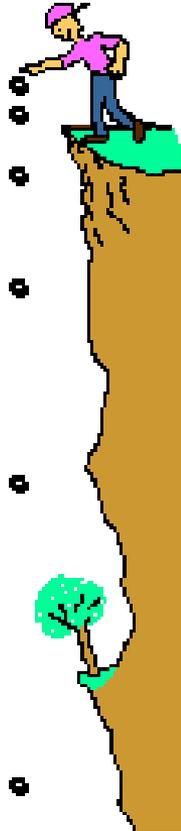
Free Fall

Any object which is moving and being acted upon only be the force of gravity is said to be "in a state of free fall."

- all objects fall freely at $g \approx 10$ m s^{-2} when near the earth and air resistance is negligible.

- speed of a free-falling body increases by 9.8 m s^{-1} every second or when a body is thrown up, its speed decreases by 9.8 m s^{-1} every second.
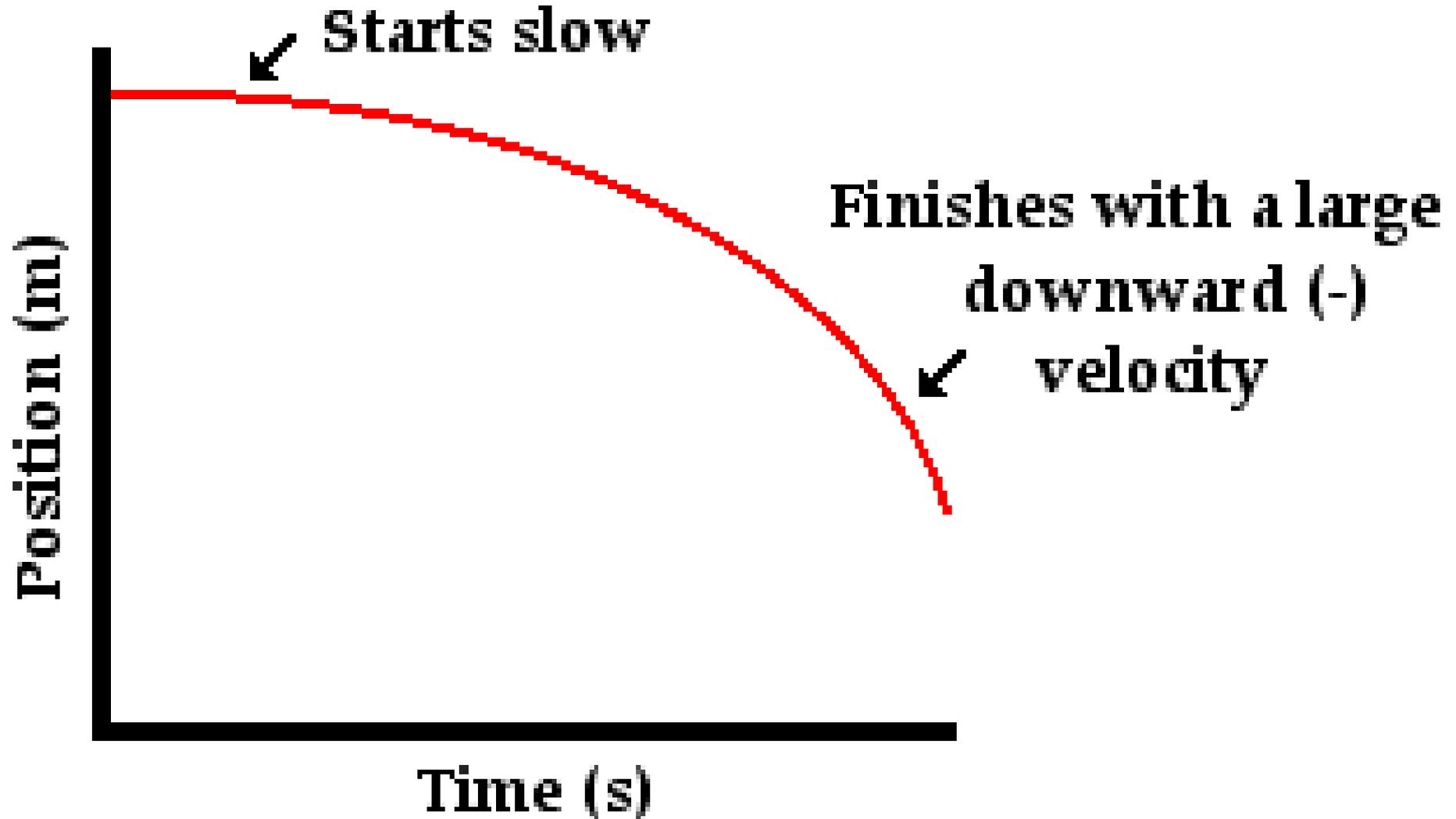
Although the acceleration due to gravity is considered constant, it tends to vary slightly over the earth since the earth is not a perfect sphere.
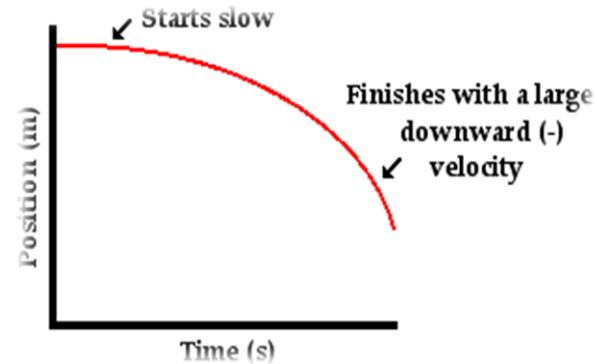
Examples

- Examples of objects in Free fall

➢ A spacecraft (in space) with its rockets off (e.g. in a continuous orbit, or going up for some minutes, and then down)

➢ The Moon orbiting around the Earth.

- Examples of objects not in Free fall

➢ Standing on the ground: the gravitational acceleration is counteracted by the normal force from the ground.

➢ Flying horizontally in an airplane: the wings' lift is also providing an acceleration.

Representing Free Fall by Graphs

Starts slow

Finishes with a large downward (-) velocity

Position (m)

Time (s)

Free fall graphs shows :

Starts slow

Finishes with a large downward (-) velocity

Position (m)

Time (s)

▶ **The line on the graph curves.**

▶ **A curved line on a position versus time graph signifies an accelerated motion.**

▶ **The position-time graph reveals that the object starts with a small velocity (slow) and finishes with a large velocity (fast).**

Check your Understanding !!

Questions to answer !

- **"Doesn't a more massive object accelerate at a greater rate than a less massive object?" "Wouldn't an elephant free-fall faster than a mouse?"**

- **The answer to the question (doesn't a more massive object accelerate at a greater rate than a less massive object?) is absolutely NOT!**

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☐**That is, absolutely not if we are considering the specific type of falling motion known as free-fall.**

☐**Free-fall is the motion of objects which move under the sole influence of gravity; free-falling objects do not encounter air resistance.**

☐**More massive objects will only fall faster if there is an appreciable amount of air resistance present.**

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Force of gravity means the dog accelerates

gravity

To start, the dog is falling slowly (it has not had time to speed up).

There is really only one force acting on the dog, the force of gravity.

The dog falls faster (accelerates) due to this force.

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Gravity is still bigger than air resistance

Air resistance

gravity

As the dog falls faster, another force becomes bigger – air resistance.

The force of gravity on the dog of course stays the same

The force of gravity is still bigger than the air resistance, so the dog continues to accelerate (get faster)

Gravity = air resistance
Terminal Velocity

Air resistance

gravity

As the dog falls faster and air resistance increases, eventually the air resistance becomes as big as (equal to) the force of gravity.

The dog stops getting faster (accelerating) and falls at constant speed.

This velocity is called the terminal Velocity

Terminal Speed
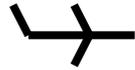
The dog will continue to fall at constant speed (called the terminal speed) until………….

Air resistance

air resistance = gravity

gravity
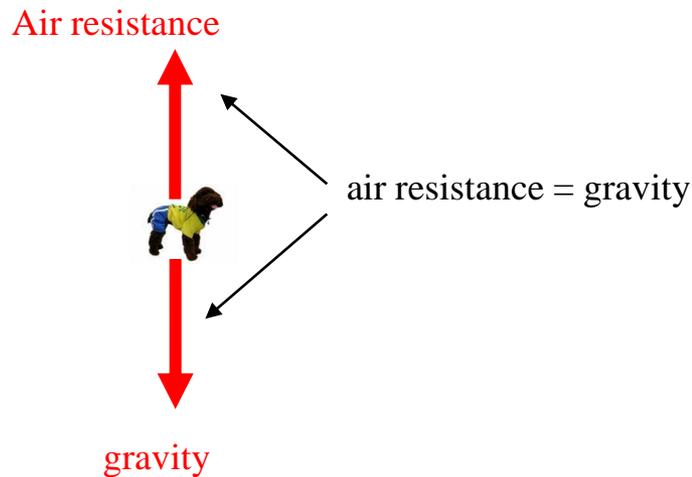
Uniformly Accelerated Motion

- Acceleration is defined as the rate of change of velocity with respect to time, in a given direction. The SI units of acceleration are ms^{-2}.

- This would mean that if an object has an acceleration of 1 ms^{-2} it will increase its velocity (in a given direction) 1 ms^{-1} every second that it accelerates.

It means that acceleration is constant.

This meaning that velocity is varying with respect to time, we see this by this formula (v - u) / t (Time).

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It means….

- If an object is held stationary in a uniform gravitational field and when it is released, it will fall. It will do so with uniform acceleration.

- Near the surface of the earth the acceleration is approximately 9.8 ms^{-2}.

- This means that every second that the object falls its velocity will increase by 9.8 ms^{-1} .

- What happens if an object is thrown up?

What happens if an object is thrown up?

- The acceleration is still downward. If an object is thrown up with an initial velocity of 30 ms^{-1}, after one second it will only be going 20 ms^{-1} up, after 2 seconds it will only be going 10 ms^{-1}, after 3 seconds the object will have zero velocity!

- Even if the objects velocity is zero the acceleration is not zero.

An experiment with 'g'.

- College building
- Stop watch
- A group of students on top floor
- A group of students on ground floor
- Need to check the distance between top floor and ground, time to calculate velocity.
- (This experiment will be carried out during next lesson)
- Upon investigation, g constant is found with one of the equations we have derived and it is as follows :

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Remember (will study in Ch.4 Dynamics)

- **The actual explanation of why all objects accelerate at the same rate involves the concepts of force and mass.**
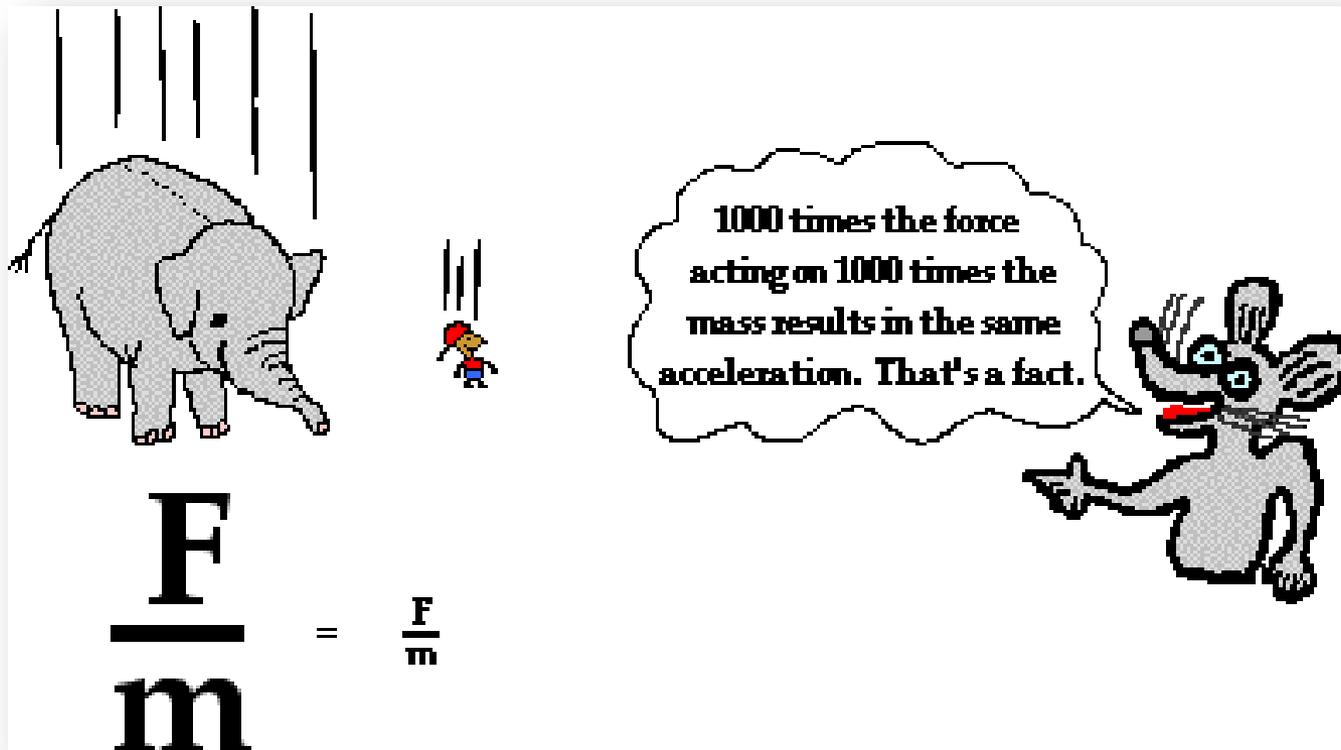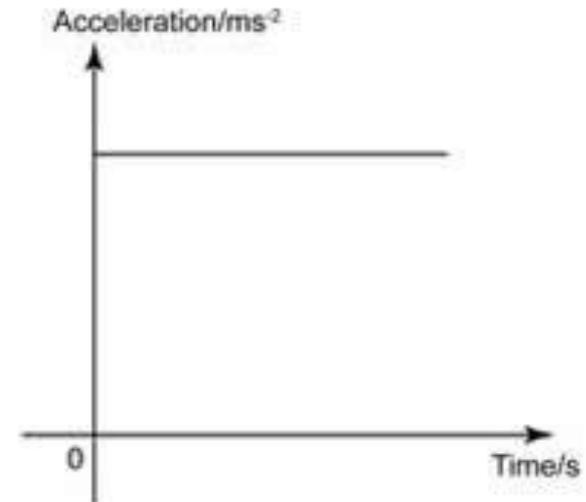
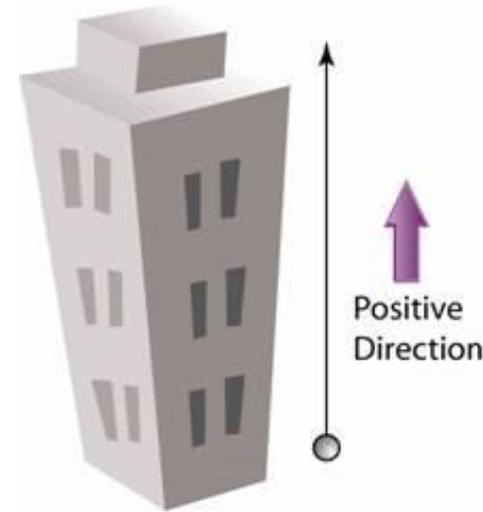Graph of free falling :
1. Dropping an object from high place

Positive Direction

• Velocity - Time Graph

Velocity/ms⁻¹

0   Time/s

• Acceleration - Time Graph

Acceleration/ms²

0   Time/s

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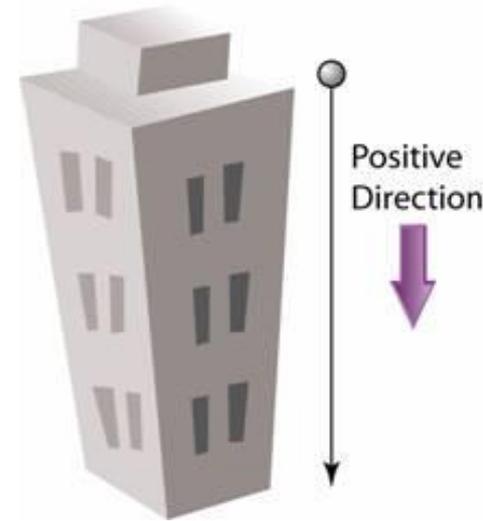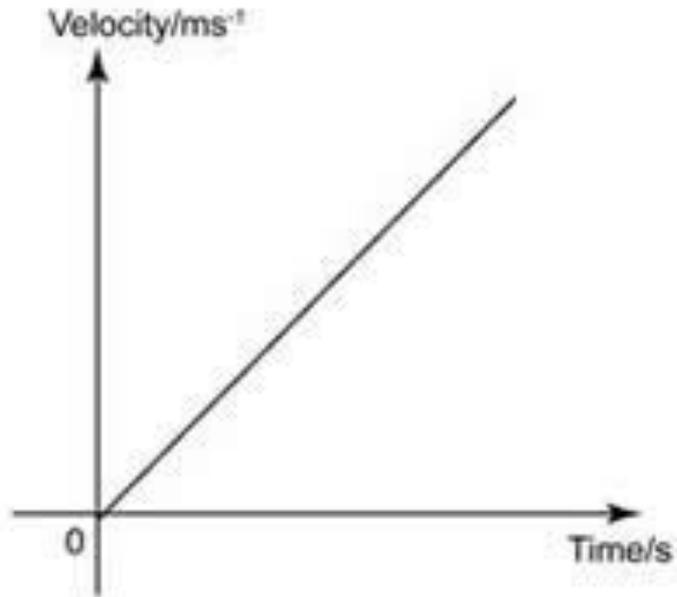
Graph of free falling :
2. **Launching Object Upward**

Positive Direction

- Velocity - Time Graph

- Acceleration - Time Graph

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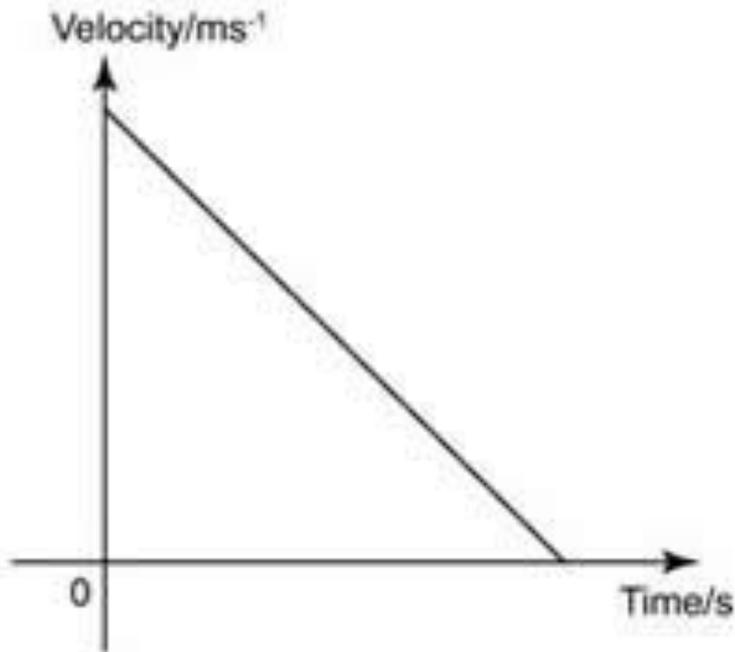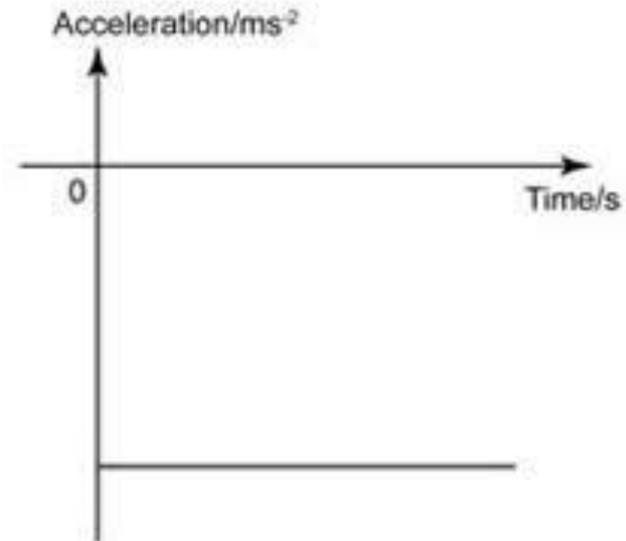
Graph of free falling :
3 . Object moving upward and fall back to the ground

Positive Direction
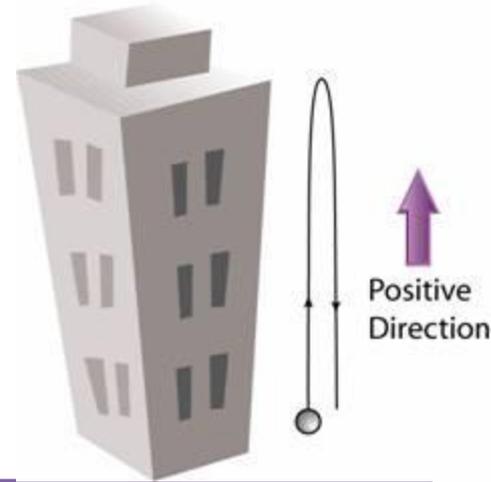
- **Velocity - Time Graph**
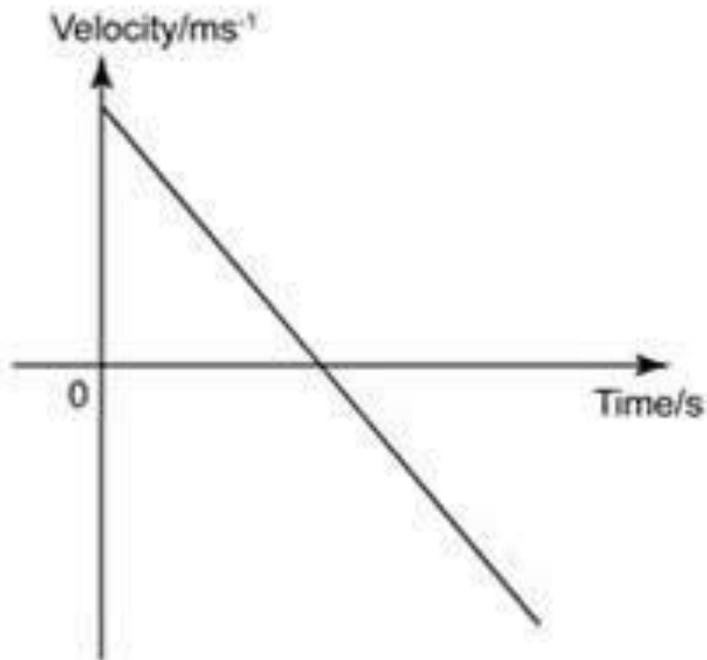
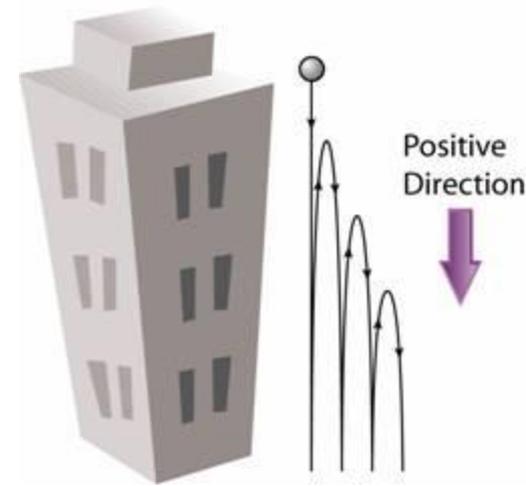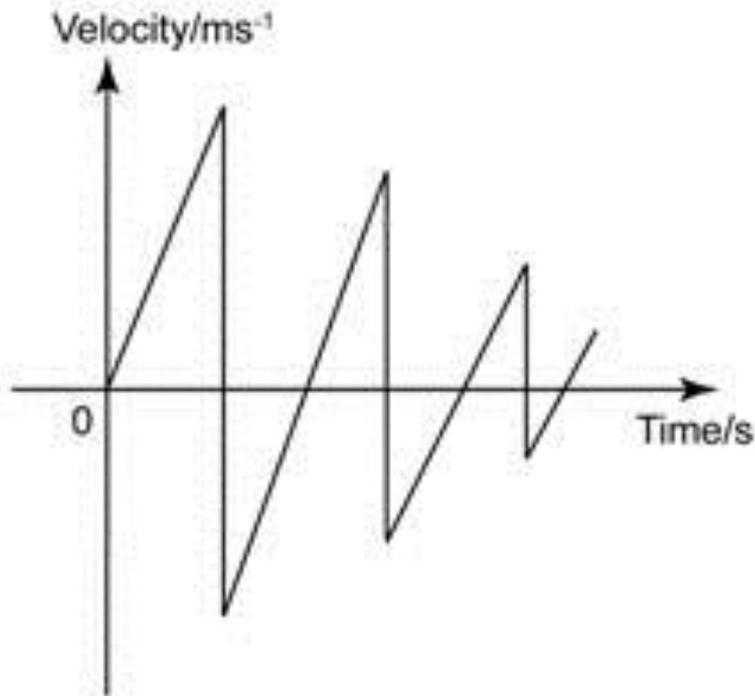- **Acceleration - Time Graph**
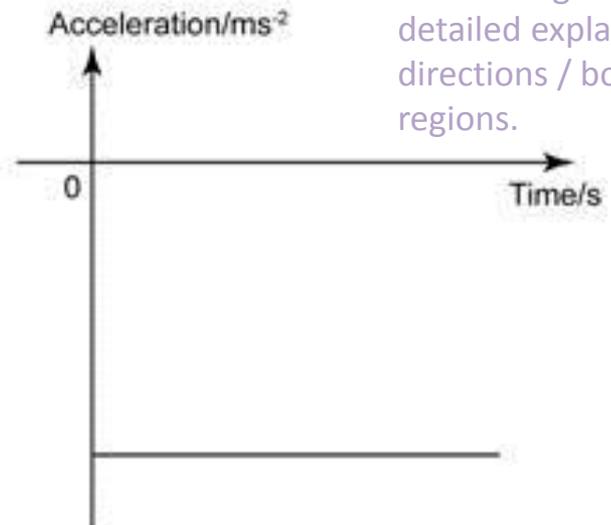
Note : The graph here is to show that the acceleration remains same. The following slides will give a detailed explanation on directions / both regions.
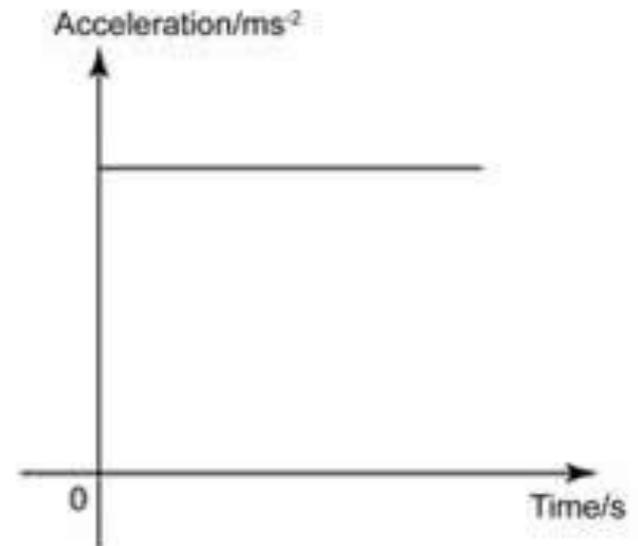

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Graph of free falling :
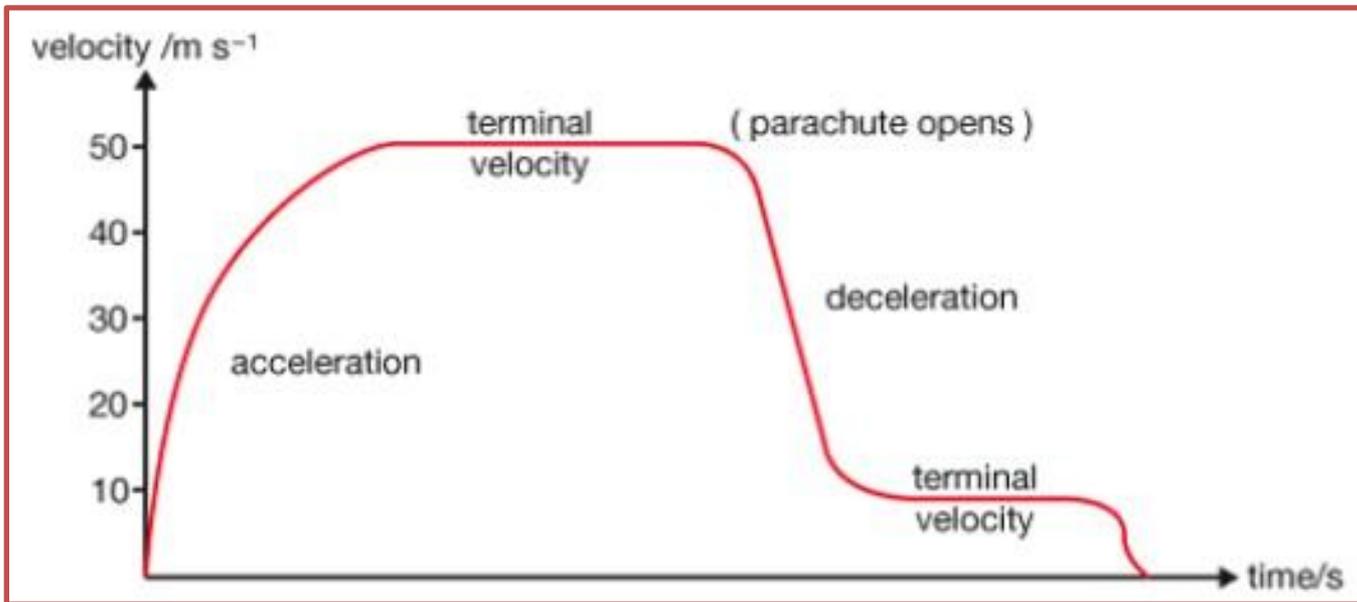## 4. Object falling and bounces back

Positive Direction

- Velocity - Time Graph

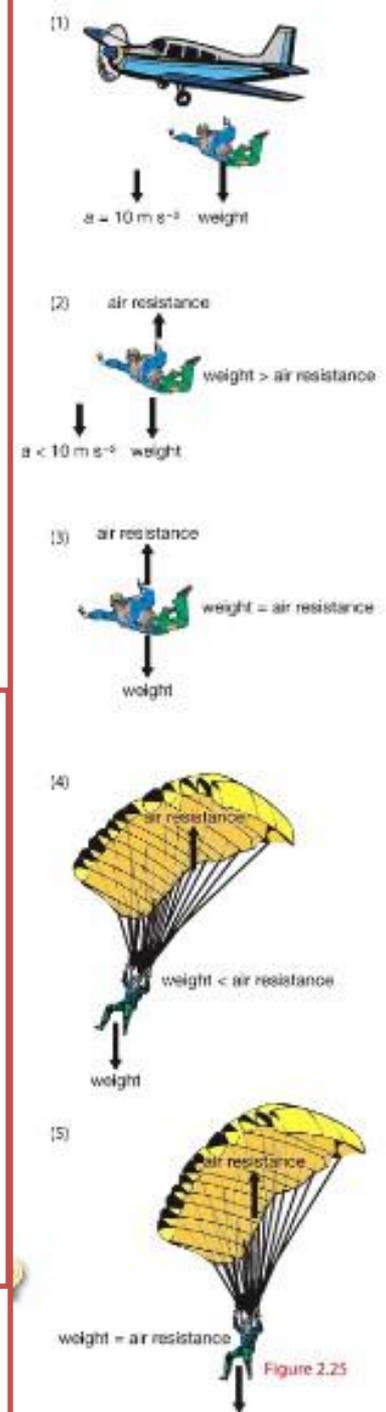

- Acceleration - Time Graph

At the point when the air resistance equals to the weight, there is no acceleration and the object will fall with "terminal velocity".

A small dense object, like a steel ball bearing, has a high terminal velocity. A light object, like a raindrop, or an object with large surface area like a piece of paper, has a low terminal velocity.
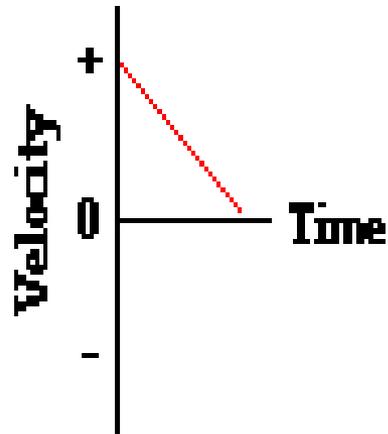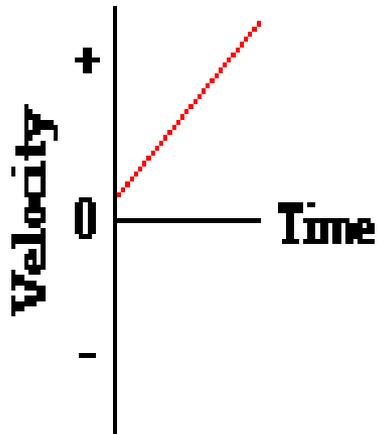
Positive Velocity & Negative Velocity

- How can one tell whether the object is moving in the positive direction (i.e., positive velocity) or in the negative direction (i.e., negative velocity)?

- And how can one tell if the object is speeding up or slowing down?
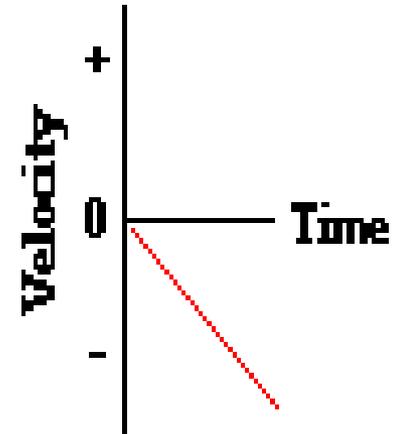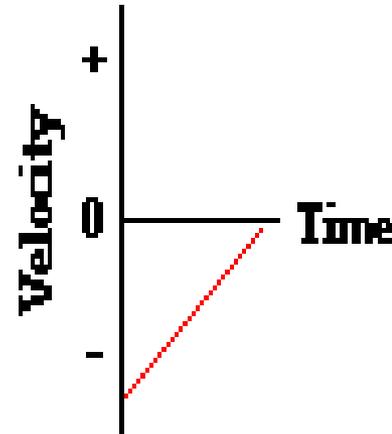
Positive Velocity & Negative Velocity

- Since the graph is a velocity-time graph, the velocity would be positive whenever the line lies in the positive region (above the x-axis) of the graph.

- Similarly, the velocity would be negative whenever the line lies in the negative region (below the x-axis) of the graph.

- A **positive velocity** means the object is moving in the positive direction; and a **negative velocity** means the object is moving in the negative direction.

- So one knows an object is moving in the positive direction if the line is located in the positive region of the graph (whether it is sloping up or sloping down). And one knows that an object is moving in the negative direction if the line is located in the negative region of the graph (whether it is sloping up or sloping down).

- And finally, if a line crosses over the x-axis from the positive region to the negative region of the graph (or vice versa), then the object has changed directions.

Positive Velocity & Negative Velocity

These objects are moving with a positive velocity.

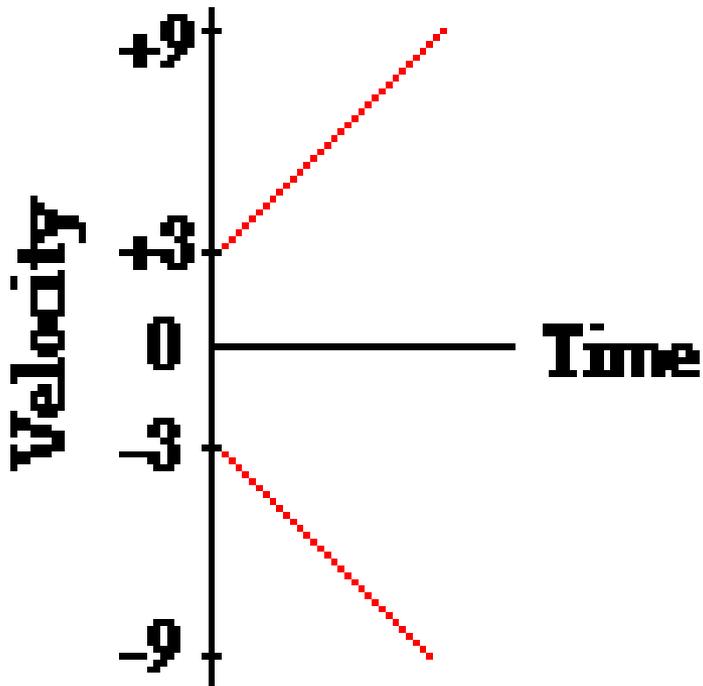These objects are moving with a negative velocity.
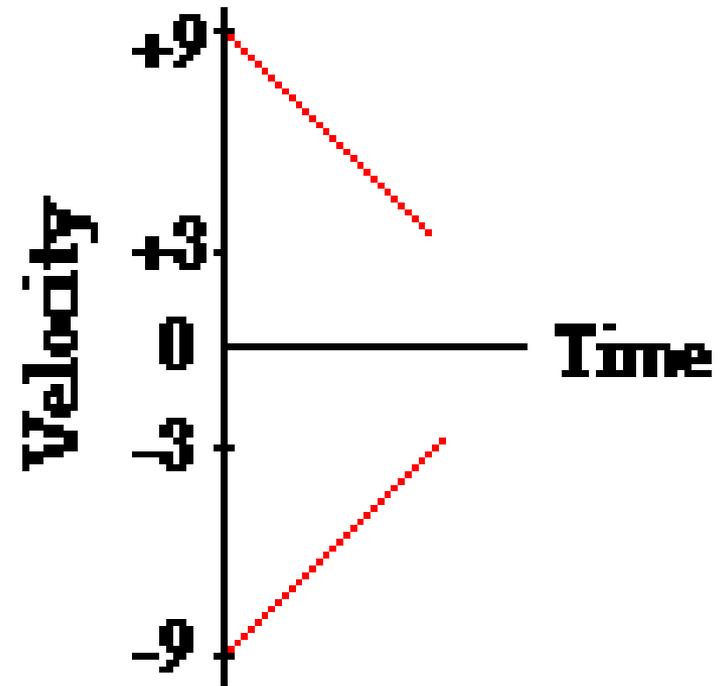
Positive Velocity & Negative Velocity

- Now how can one tell if the object is speeding up or slowing down?

- Speeding up means that the magnitude of the velocity is getting large. For instance, an object with a velocity changing from +3 m/s to + 9 m/s is speeding up. Similarly, an object with a velocity changing from -3 m/s to -9 m/s is also speeding up.

- In each case, the magnitude of the velocity is increasing; the speed is getting bigger.

- Given this fact, one would believe that an object is speeding up if the line on a velocity-time graph is changing from near the 0-velocity point to a location further away from the 0-velocity point. That is, if the line is getting further away from the x-axis (the 0-velocity point), then the object is speeding up. And conversely, if the line is approaching the x-axis, then the object is slowing down.

Imran Mirza (MSc Physics. PGCC University of StrethClyde, Scotland, UK)

Positive Velocity & Negative Velocity

Speeding Up

+9
+3
Velocity
0 ———— Time
-3
-9

Slowing Down

+9
+3
Velocity
0 ———— Time
-3
-9

Imran Mirza (MSc Physics. PGCC University of StrethClyde, Scotland, UK)

Equations of Motion

There are 4 equations that you can use whenever an object moves with **_constant, uniform acceleration in a straight line_**. The equations are written in terms of the 5 symbols in the box:

s = displacement (m)

u = initial velocity (ms^{-1})

v = final velocity (ms^{-1})

a = constant acceleration (ms^{-2})

t = time interval (s)

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Since a = (v - u) / t

$$v = u + at \quad \ldots (1)$$

If acceleration is constant, the average velocity during the motion will be half way between v and u. This is equal to ½(u + v).

$$\tfrac{1}{2}(u + v) = s/t$$

$$s = \tfrac{1}{2}(u + v)t \quad \ldots (2)$$

Using equation (1) to replace v in equation (2):

$$s = \tfrac{1}{2}(u + u + at)t$$

$$s = \tfrac{1}{2}(2u + at)t$$

$$s = ut + \tfrac{1}{2}at^2 \quad \ldots (3)$$

From equation (1), t = (v – u)/a

Using this to replace t in equation (2):

$s = ½(u + v)[(v - u)/a]$

$2as = (u + v)(v – u)$

$2as = v^2 – u^2$

$v^2 = u^2 + 2as$ … (4)

Note:

• You can only use these equations only if the acceleration is constant.

• Notice that each equation contains only 4 of our 5 "s, u, v, a, t" variables. So if know any 3 of the variables, we can use these equations to find the other 2.

Example 4

A cheetah starts from rest and accelerates at 2.0 ms⁻² due east for 10 s. Calculate (a) the cheetah's final velocity, (b) the distance the cheetah covers in this 10 s.

Solution:

(a) Using equation (1): $v = u + at$

$$v = 0 + (2.0 \text{ ms}^{-2} \times 10 \text{ s}) = 20 \text{ ms}^{-1} \text{ due east}$$

(b) Using equation (2): $s = \frac{1}{2}(u + v)t$

$$s = \frac{1}{2}(0 + 20 \text{ ms}^{-1}) \times 10 \text{ s} = 100 \text{ m due east}$$

You could also find the displacement by plotting a velocity-time graph for this motion. The magnitude of the displacement is equal to the area under the graph.

Imran Mirza (MSc Physics. PGCC University of StrethClyde, Scotland, UK)

Example 5

An athlete accelerates out of her blocks at 5.0 ms^{-2}. (a) How long does it take her to run the first 10 m? (b) What is her velocity at this point?

Solution:

(a) Using equation (3): $s = ut + \frac{1}{2}at^2$

$$10 \text{ m} = 0 + (1/2 \times 5.0 \text{ ms}^{-2} \times t^2)$$

$$t^2 = 4.0 \text{ s}^2$$

$$t = 2.0 \text{ s}$$

(b) Using equation (1): $v = u + at$

$$v = 0 + (5.0 \text{ ms}^{-2} \times 2.0 \text{ s})$$

$$v = 10 \text{ ms}^{-1}$$

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Example 6

A bicycle's brakes can produce a deceleration of 2.5 ms^{-2}. How far will the bicycle travel before stopping, if it is moving at 10 ms^{-1} when the brakes are applied?

Solution:

Using equation (4): $v^2 = u^2 + 2as$

$$0 = (10 \text{ ms}^{-1})^2 + (2 \times (-2.5 \text{ ms}^{-2}) \times s)$$

$$0 = 100 \text{ m}^2\text{s}^{-2} - (5.0 \text{ ms}^{-2} \times s)$$

$$s = 20 \text{ m}$$

Example 7

A student flips a coin into the air. Its initial velocity is 8.0 ms^{-1}. Taking g = 10 ms^{-2} and ignoring air resistance, calculate: (a) the maximum height, h, the coin reaches, (b) the velocity of the coin on returning to his hand, (c) the time that the coin is in the air.

Solution: (upward motion to be negative)

(a) $v^2 = u^2 + 2as$

$0 = (8.0 \text{ ms}^{-1})^2 + (2 \times (-10 \text{ms}^{-2}) \times h)$

h = 3.2 m

(b) The acceleration is the same going up and coming down. If the coin decelerates from 8.0 ms^{-1} to 0 ms^{-1} on the way up, it will accelerate from 0 ms^{-1} to 8 ms^{-1} on the way down. The motion is symmetrical. So the velocity on returning to his hand is 8.0 ms^{-1} downwards.

(c) $v = u + at$

$0 = 8.0 \text{ ms}^{-1} + (-10 \text{ ms}^{-2} \times t)$

t = 0.8 s

The coin will take the same time between moving up and coming down. So total time in the air = 1.6 s.

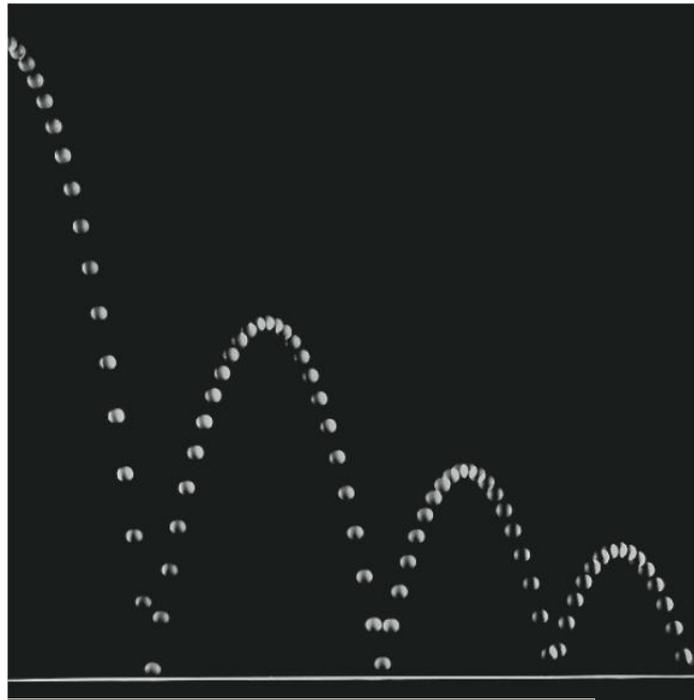# You-tube videos links with explanation on : Newtonian Mechanism - Kinematics

- http://www.youtube.com/watch?v=go9uekKOcKM

- http://www.youtube.com/watch?v=xE71aKXjss0&feature=related

3. Kinematics

(k) describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

PROJECTILE is a body which is thrown horizontally or at an angle relative to the horizontal which follows a curved path called trajectory.

A projectile is an object moving in two dimensions under the influence of Earth's gravity; its path is a parabola
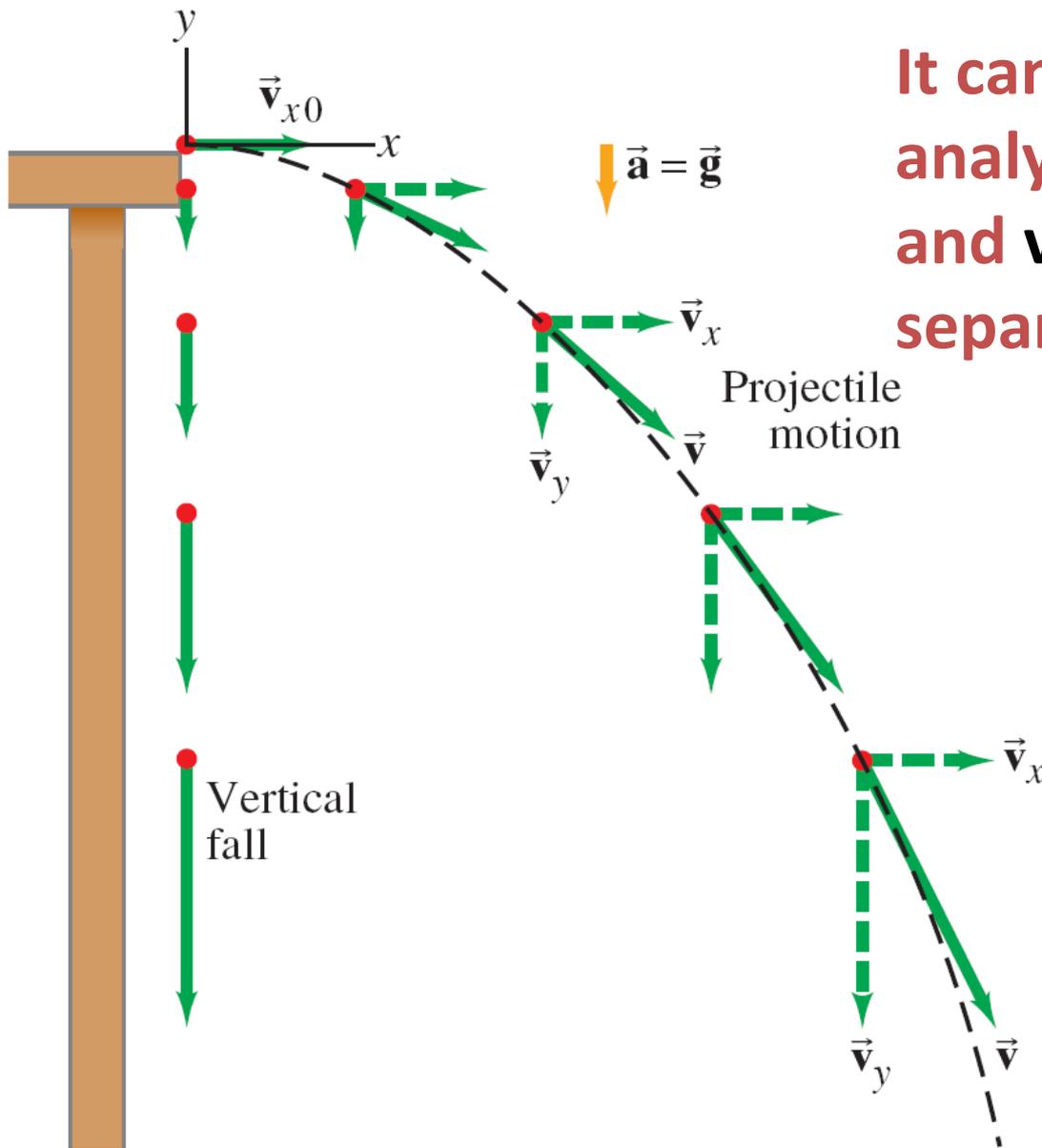
Examples:
Ball being thrown, water coming out of the hose, a bullet fired from a gun, arrow shot from a bow, fountains.

Imran Mirza (MSc Physics. PGCC University of StrethClyde, Scotland, UK)

What is projectile motion?

*Made up of **horizontal** and **vertical** components

*Movement wherein an object is acted upon by gravity and air resistance

*Motion of a body following a curved path

Imran Mirza (MSc Physics. PGCC University of StrethClyde, Scotland, UK)

PARABOLIC

MOTION OF

PROJECTILE

Imran Mirza (MSc Physics. PGCC University of StrethClyde, Scotland, UK)

It can be understood by analyzing the **horizontal** and **vertical** motions separately.
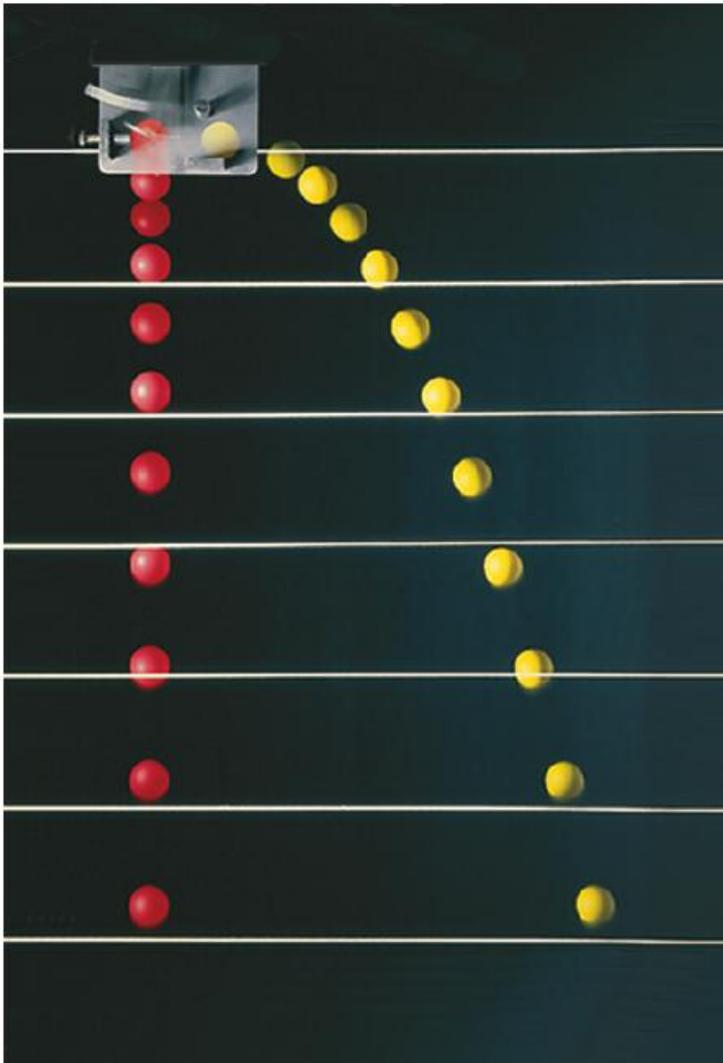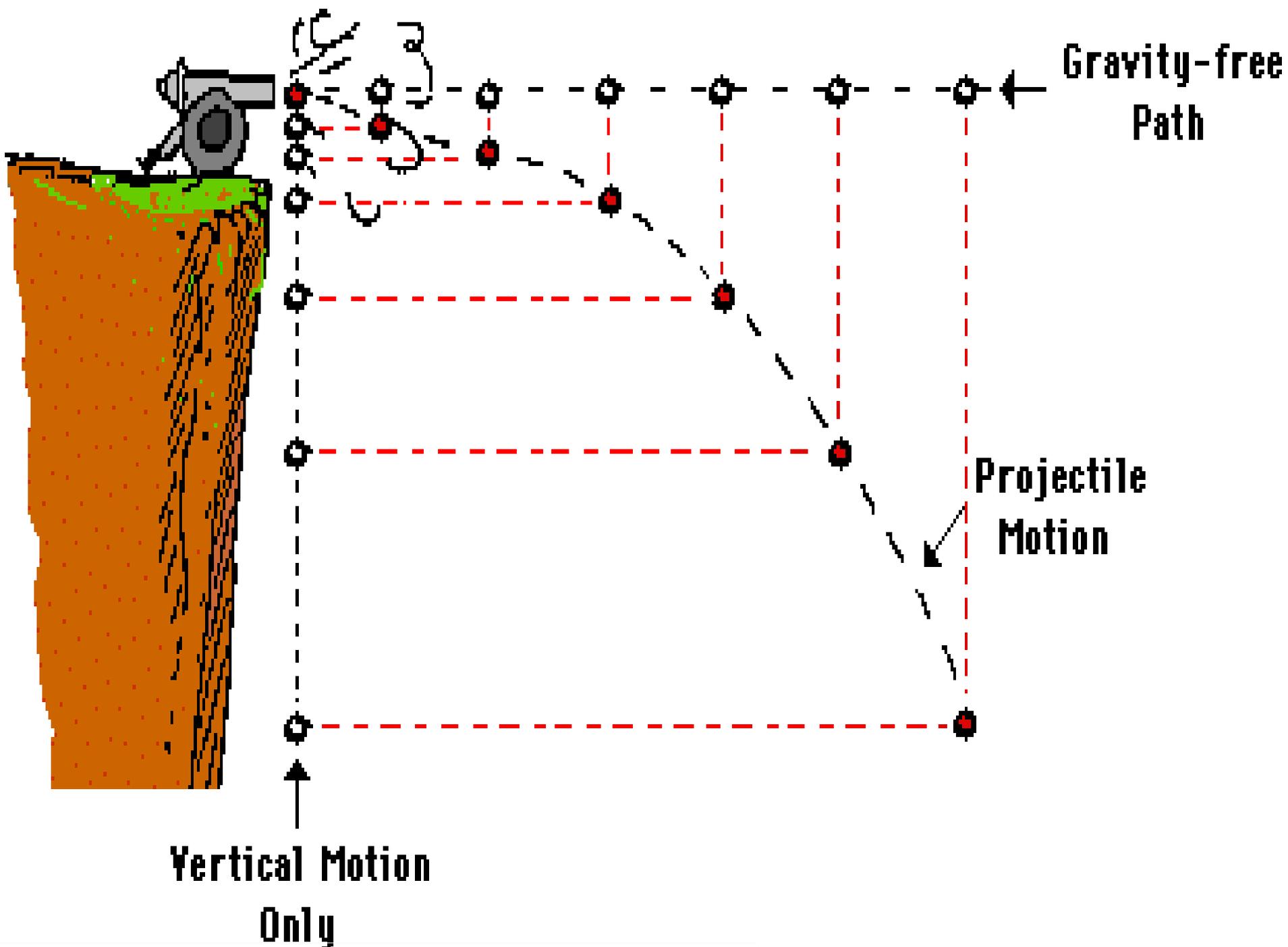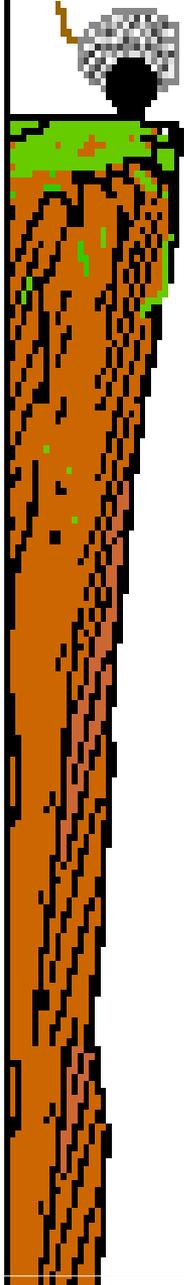
The speed in the *x*-direction is constant; in the *y*-direction the object moves with constant acceleration *g*.

This photograph shows two balls that start to fall at the same time. The one on the right has an initial speed in the *x*-direction. It can be seen that vertical positions of the two balls are identical at identical times, while the horizontal position of the yellow ball increases linearly.

Imran Mirza (MSc Physics. PGCC University of StrethClyde, Scotland, UK)

REMEMBER:

1. The horizontal velocity of a projectile is <u>*constant*</u> (never changing in value),

2. There is uniform vertical acceleration <u>*caused by gravity*</u>; its value is 9.8 m/s^2

3. The vertical velocity of a projectile changes by ~10 m/s each second,
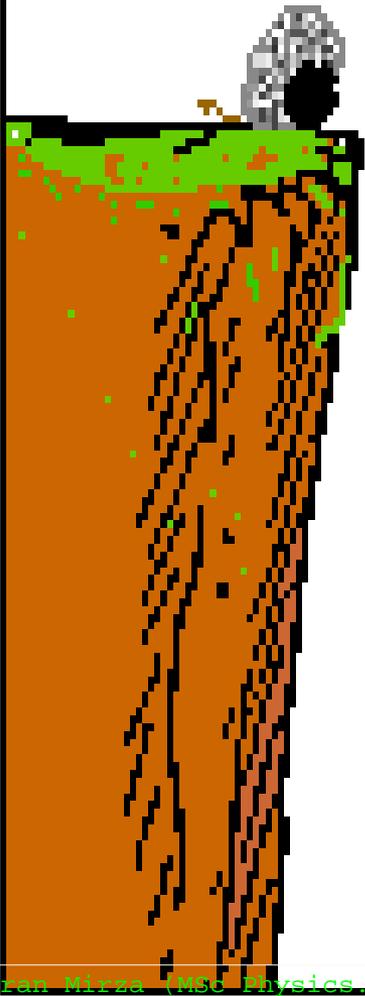the horizontal motion of a projectile is <u>*independent*</u> of its vertical motion.

Gravity-free
Path

Projectile
Motion

Vertical Motion

Only

PROJECTILE I

(SIMPLE PROJECTILE)

$$t = \; -- \; s$$

$$v_x = \; -- \; m/s \quad v_y = \; -- \; m/s$$

Imran Mirza (MSc Physics, PGCC University of StrethClyde, Scotland, UK)

PROJECTILE II (WITH ANGLE)

$$t = \ -- \ s$$

$$v_x = \ -- \ m/s \quad v_y = \ -- \ m/s$$

*All vertical components have y subscripts :
vy, dy, ty,
*All horizontal components have x subscripts:
vx, dx

RANGE is the horizontal displacement of the projectile (dx)

MAXIMUM HEIGHT is the vertical displacement of the projectile (dy)

How to calculate – Projectile Motion
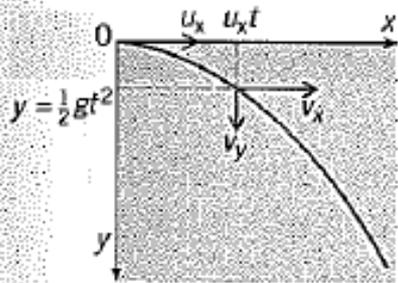## (From your reference book)

Figure 3.18

This topic is often called **projectile motion**. Galileo first gave an accurate analysis of this motion. He did so by splitting the motion up into its vertical and horizontal components, and considering these separately. The key is that the two components can be considered independently.

As an example, think about a particle sent off in a horizontal direction and subject to a vertical gravitational force (its weight). As before, air resistance will be neglected. We will analyse the motion in terms of the horizontal and vertical components of velocity. The particle is projected at time $t = 0$ at the origin of a system of x, y co-ordinates (Figure 3.18) with velocity u_x in the x-direction. Think first about the particle's vertical motion (in the y-direction). Throughout the motion, it has an acceleration of g (the acceleration of free fall) in the y-direction. The initial value of the vertical component of velocity is $u_y = 0$. The vertical component increases continuously under the uniform acceleration g. Using $v = u + at$, its value v_y at time t is given by $v_y = gt$. Also at time t, the vertical displacement y downwards is given by $y = \frac{1}{2}gt^2$. Now for the horizontal motion (in the x-direction): here the acceleration is zero, so the horizontal component of velocity remains constant at u_x. At time t the horizontal displacement x is given by $x = u_x t$. To find the velocity of the particle at any time t, the two components v_x and v_y must be added vectorially. The direction of the resultant vector is the direction of motion of the particle. The curve traced out by a particle subject to a constant force in one direction is a **parabola**.

If an object is launched at an initial angle of ϑ_0 with the horizontal, the analysis is similar except that the initial velocity has a vertical component.
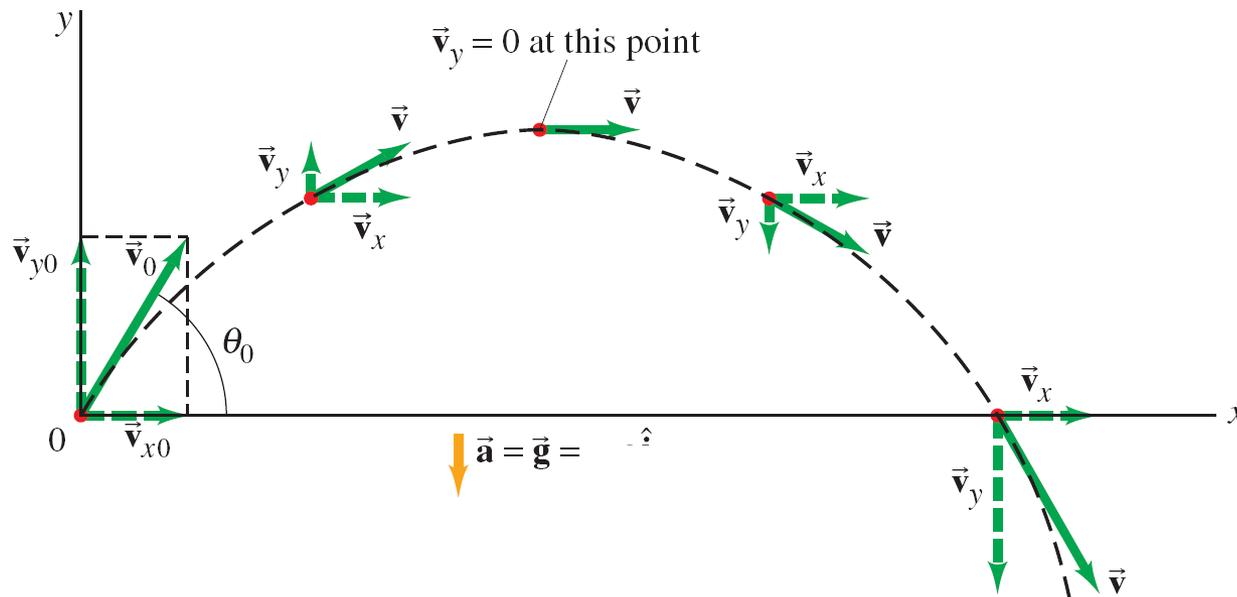
Path of a projectile fired with initial velocity v_0 at angle θ_0 to the horizontal. Path is shown dashed in black, the velocity vectors are green arrows, and velocity components are dashed. The acceleration $a = dv/dt$ is downward. That is, $a = g$.

Imran Mirza (MSc Physics. PGCC University of StrethClyde, Scotland, UK)

(Not in syllabus, only for your info)
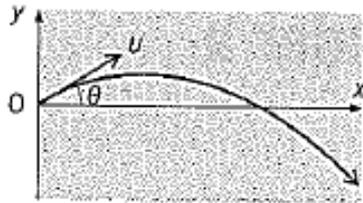

Figure 3.20

Figure 3.21

If the particle had been sent off with velocity u at an angle θ to the horizontal, as in Figure 3.20, the only difference to the analysis of the motion is that the initial y-component of velocity is $u \sin \theta$. In the example illustrated in Figure 3.20 this is upwards. Because of the downwards acceleration g, the y-component of velocity decreases to zero, at which time the particle is at the crest of its path, and then increases in magnitude again but this time in the opposite direction. The path is again a parabola.

For the particular case of a particle projected with velocity u at an angle θ to the horizontal from a point on level ground (Figure 3.21), the range R is defined as the distance from the point of projection to the point at which the particle reaches the ground again. We can show that R is given by
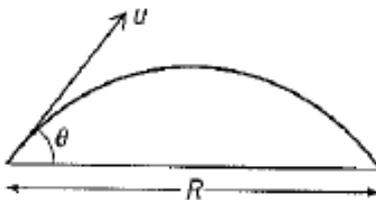
$$R = \frac{(u^2 \sin 2\theta)}{g}$$

Sample Problem – Projectile Motion

A stone is thrown from the top of a vertical cliff, 45 m high above level ground, with an initial velocity of 15 m s^{-1} in a horizontal direction (Figure 3.22). How long does it take to reach the ground? How far from the base of the cliff is it when it reaches the ground?
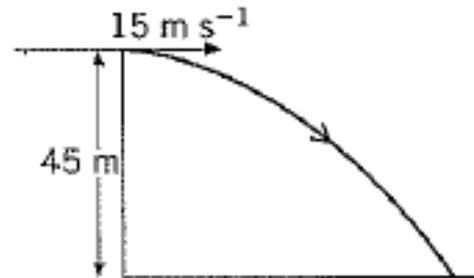

Figure 3.22

To find the time t for which the stone is in the air, work with the vertical component of the motion, for which we know that the initial component of velocity is zero, the displacement $y = 45$ m, and the acceleration a is 9.81 m s^{-2}. The equation linking these is $y = \frac{1}{2}gt^2$. Substituting the values, we have $45 = \frac{1}{2} \times 9.81t^2$. This gives $t = \sqrt{(2 \times 45/9.81)} = \mathbf{3.0\,s}$.

For the second part of the question, we need to find the horizontal distance x travelled in the time t. Because the horizontal component of the motion is not accelerating, x is given simply by $x = u_x t$. Substituting the values, we have $x = 15 \times 3.0 = \mathbf{45\,m}$.

Any Questions?